**Math 3 - Discrete math (V3020) Exercises, week 10 (due Nov 11th, 2025)**

1. Let $G = (V, T, S, P)$ be the phrase-structure grammar with the vocabulary $V = \{0, 1, A, S\}$, the terminals $T = \{0, 1\}$, and the set of productions $P$ consisting of $S \to 1S$, $S \to 00A$, $A \to 0A$, and $A \to 0$. $S$ is the start symbol.

   (a) Show that 111000 belongs to the language generated by $G$.

   (b) Show that 11001 does not belong to the language generated by $G$.

   (c) What is the language generated by G?

2. Construct phrase-structure grammars to generate the following sets.

   (a) $\{1^n \mid n \geqslant 0\}$

   (b) $\{10^n \mid n \geqslant 0\}$

   (c) $\{(11)^n \mid n \geqslant 0\}$

3. Draw the state diagram for the finite-state machine with the following state table.

| | $f$ | | $g$ | |
|---|---|---|---|---|
| | Input | | Output | |
| State | 0 | 1 | 0 | 1 |
| $s_0$ | $s_1$ | $s_0$ | 0 | 0 |
| $s_1$ | $s_2$ | $s_0$ | 1 | 1 |
| $s_2$ | $s_0$ | $s_3$ | 0 | 1 |
| $s_3$ | $s_1$ | $s_2$ | 1 | 0 |

   What is the output generated by the input string 01110? $s_0$ is the initial state.

4. Determine whether each of these strings is recognized by the deterministic finite-state automaton in Fig. 1 (on the next page).

   (a) 1101

   (b) 0101010

5. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that end with 10.

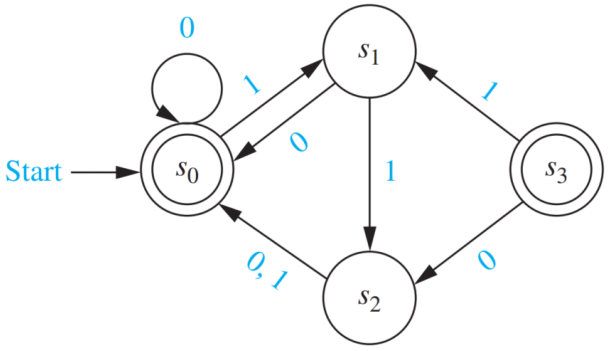


Figure 1: The state diagram for problem 4.