Math 3 - Discrete math (V3020) Exercises, week 6 (due Oct 14th, 2025)

1. Prove the "$\Leftarrow$" direction of Theorem 3.2.3, i.e. the following statement. If every cycle of graph G has an even length, then G is a bipartite graph.
2. Are the following graphs Eulerian or semi-Eulerian? Why?
   (a) The complete graph K_5,
   (b) the complete bipartite graph $K_{2,3}$.
3. Use Fleury's algorithm to find an Eulerian trail for the graph in Fig. 1
4. Are the following graphs Hamiltonian or semi-Hamiltonian? Why?
   (a) The complete graph K_5,
   (b) the complete bipartite graph $K_{2,3}$.
5. Study Chapter 3.2.4 and pick an application that you are most interested in (the shortest path problem, the Chinese postman problem, or the traveling salesman problem). Come up with a simple graph and solve your chosen problem for it. These are very common applications of graph theory, and are especially good practice for those who have chosen to do their home exam project on the topic.

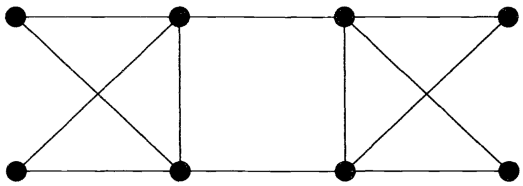

Figure 1: The graph for problem 3.