Math 3 - Discrete math (V3020) Exercises, week 9 (due Nov 4th, 2025)

1. Are the following sets, equipped with the specified binary operations, groups? Why/why not?
   (a) The even integers, equipped with the "standard" addition?
   (b) The odd integers, equipped with the "standard" addition?
2. Show that the set $\{a, b, c, e\}$ is a group when equipped with $\oplus$ that operates according to the following table.

$\oplus$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	a	e
c	c	b	e	a

3. The set of complex numbers $\{1, -1, i, -i\}$, equipped with multiplication, is a group. Show that it is isomorphic to the group in Problem 2. Recall that $i^2 = i \cdot i = -1$.
4. Show that the set of all rational numbers $\mathbb{Q} = \left\{ \frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{Z} \setminus \{0\} \right\}$ is a field when equipped with the "standard" addition and multiplication operations.
5. Factorize 26149 using Pollard's $p - 1$ method.