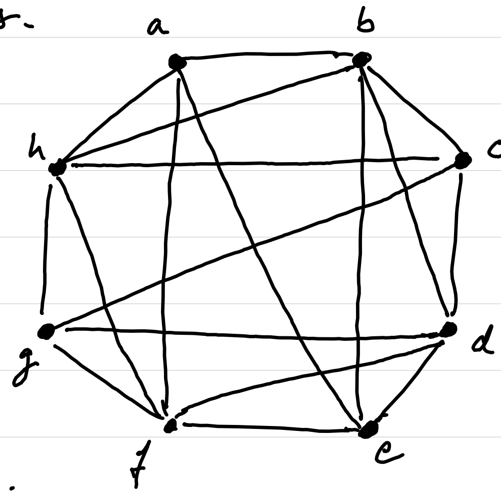
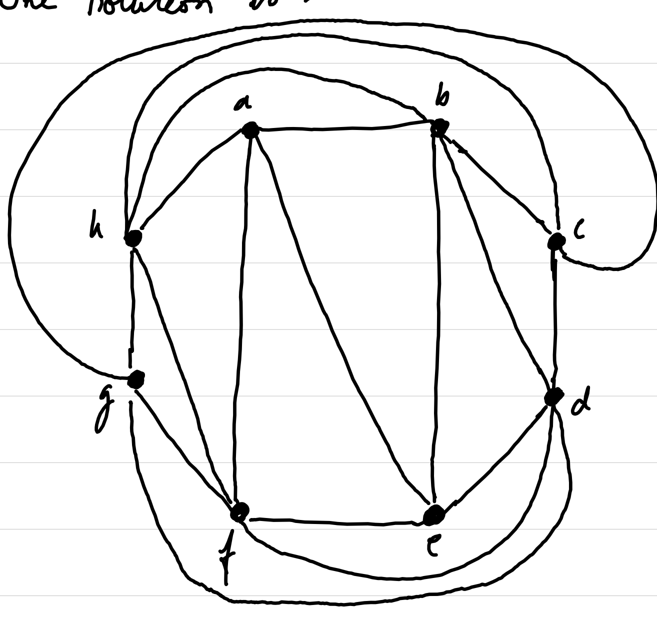
Math 3 (V3020) Exercise 8

1. Show that the following graph can be drawn in the plane without crossings.

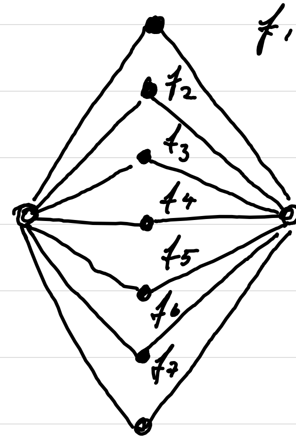


One solution is:



(2.) Verify Euler's formula for the complete bipartite graph $K_{2,7}$.

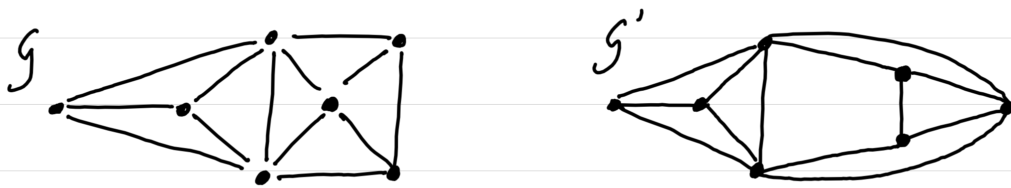
We draw $K_{2,7}$ so that we can easily count the vertices, edges, and faces



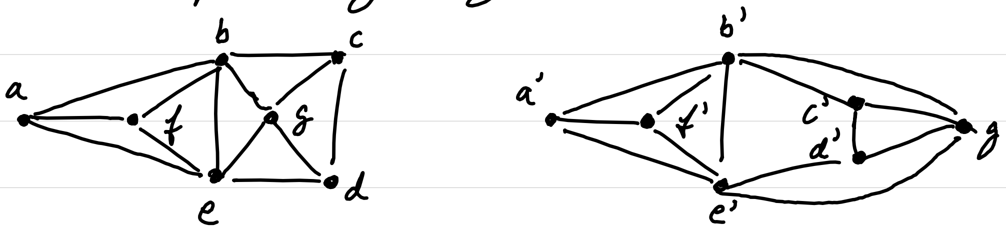
Clearly there are $n = 2 + 7 = 9$ vertices with $m = 7 + 7 = 14$ edges
The $k = 7$ faces (with f_1 being an infinite face) have been marked on the drawing.

We can indeed verify
$n - m + k = 9 - 14 + 7 = 2$,
so Euler's formula holds for $K_{2,7}$ as it should because we had a plane drawing of a simple connected graph.

3. Show that the following graphs are isomorphic, but that their geometric duals are not isomorphic.



Let us label the vertices of G and G' in the following way

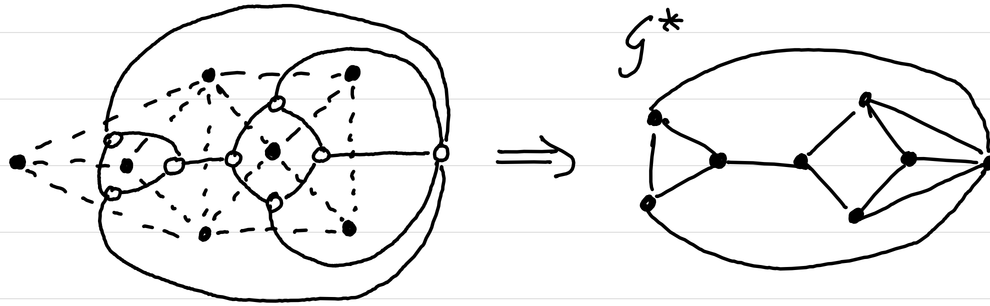


and draw the correspondence $a \longleftrightarrow a'$, $b \longleftrightarrow b'$, …, $g \longleftrightarrow g'$.

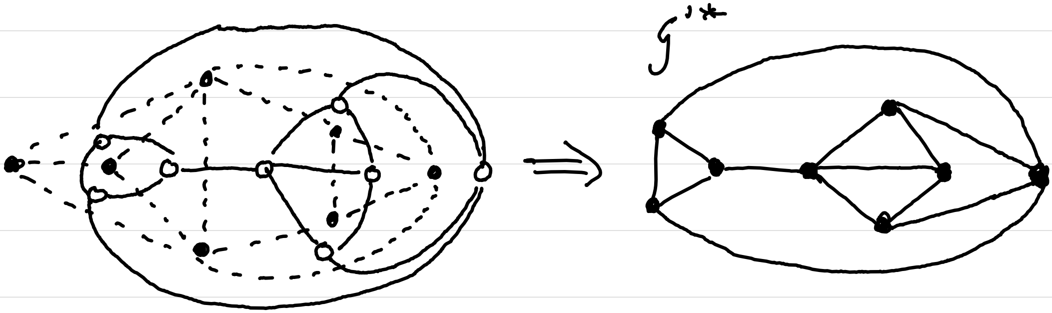
We see that the number of edges joining any two vertices in G is the same as the number of vertices joining the corresponding vertices in G'. Thus G and G' are isomorphic.

Let us form the geometric duals

G^*



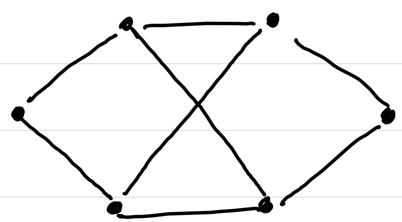
G'^*



We notice that G^* has a single vertex with degree 5. There is no such vertex in G'^*. G^* and G'^* cannot thus be isomorphic.

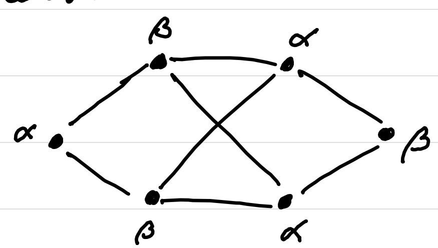
4. Find the chromatic number of

a)



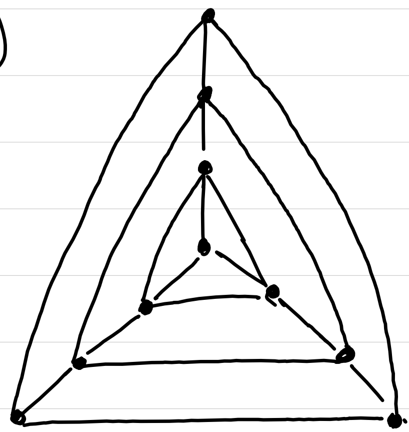
We need to find $k \in \mathbb{N}$ for which this graph is k-colorable but not $k-1$ colorable. Being k-colorable means we can color all vertices with k color so that adjacent vertices have different colors.

We use Greek letters to denote different colors.

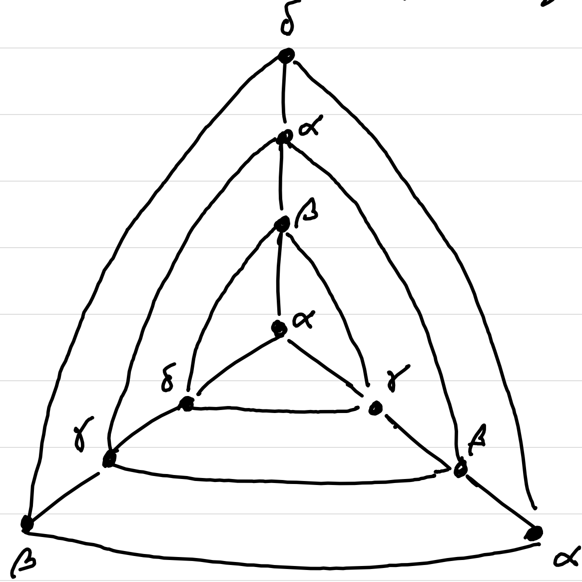


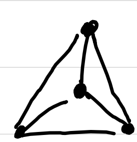
This shows that the graph is 2-colorable. Clearly it is not 1 colorable so the chromatic number is 2.

b)



We find the following 4-coloring



Just looking at the "middle" subgraph , we see that it is not 3-colorable.

$\Longrightarrow$ The chromatic number of the graph is 4.